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Stability of the ion-temperature-gradient-driven mode with negative magnetic shear

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A model for transition to the enhanced reverse shear or negative central shear mode triggered in tokamaks is proposed. This model takes into account the linear behavior of the ion temperature gradient (ITG) driven perturbation, considered nowadays as the dominant source of anomalous energy losses in the low confinement mode, in the presence of a radially varying parallel velocity. Analytic and numerical studies show that when the magnetic shear has the same sign as the second derivative of the parallel velocity with respect to the radial coordinate, the ITG mode may become more unstable. On the other hand, when the magnetic shear has the opposite sign to the second derivative of the parallel velocity, the linear ITG mode may be completely stabilized. This result is similar to our earlier works on parallel velocity shear instability [S. Sen *et al.*, Phys. Plasmas **7**, 1192 (2000); D. R. McCarthy *et al.*, Phys. Plasmas **8**, 3645 (2001)]. © *2003 American Institute of Physics.* [DOI: 10.1063/1.1616015]

I. INTRODUCTION

In order for the tokamak to become a leading contender for a fusion reactor it should develop a magnetic configuration that has good confinement and stability and a large fraction of bootstrap current. Understanding and control of turbulent transport and of its underlying driving agents is therefore a prerequisite in this process. Recent discoveries of various enhanced performance operational regimes like the H-modes,[1] the VH-modes,[2] the enhanced reverse shear modes (ERS-modes) (Ref. 3) or the negative central magnetic shear (NCS-modes) (Ref. 4) the radiative improved modes (RI-modes) (Ref. 5) has opened up a new window for improved tokamak operation.

An important challenge for enhanced tokamak operation is the development and understanding of the basic physics involved in the process that leads to the transition to the improved confinement modes. While a sheared poloidal (toroidal) flow is found to be responsible for the H- (VH-) modes, a hollow q profile (hence normally a hollow current profile) is necessary for the ERS or NCS modes. Most tokamaks, however, operate with inductive current drive, which in general produces a peaked current density profile at the magnetic axis because of the strong dependence of the plasma conductivity on the electron temperature. Only by noninductive current drive or transient techniques can a hollow current density profile be generated.

All these improved modes in the tokamak *core* seem to have a common feature that the formation of the transport barrier is usually accompanied by a jump in the toroidal velocity in the region where the transport barrier is formed. Although in the beginning of the ERS plasma in the Tokamak Fusion Test Reactor (TFTR) usually a balanced injection was used resulting in almost no toroidal flow, in more recent shots using different applied torques from the neutral beam injection (NBI) it has been confirmed that the toroidal velocity also plays an important role in the TFTR ERS-mode.[6] Similarly, RI-modes experiments in the TEXTOR-94 indicate that the toroidal flow may also play a crucial role in the transition to the RI-mode.[7] It is usually believed that the (sheared) toroidal velocity gives rise to a (sheared) radial electric field (**E**) and thereby (by the **E**×**B** shear) suppressing fluctuations and improving the core confinement. However, while this **E**×**B** shear stabilization mechanism *alone* can satisfactorily explain the confinement improvement in the edge, it may not be an obvious explanation for the core confinement improvement. This is because from the radial force balance equation,

$$v_{\perp}\approx E/B_{\phi}=\frac{v_{\phi}B_{\theta}}{B_{\phi}}-v_{\theta}+\frac{\partial P_i/\partial r}{nZeB_{\phi}},$$

it is obvious that with $B_{\phi}\gg B_{\theta}$, v_{ϕ} can contribute only weakly to $v_{\perp}$. As a result to produce the same change in E,

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v_ϕ should change by $(B_\phi/B_\theta)(\gg 1)$ times that of v_ϕ. Experiments, however, indicate otherwise. It is also supported by the fact that the formation of the ERS mode can occur at values of $\gamma_{\mathbf{E}\times\mathbf{B}}$ ($\mathbf{E}\times\mathbf{B}$ shearing rate) as much as a factor of 3 below γ_{MAX} (the maximum linear growth rate), while for the suppression of turbulence-induced transport the condition $\gamma_{\mathbf{E}\times\mathbf{B}} \geqslant \gamma_{\mathrm{MAX}}$ need to be satisfied.[8]

In this work, we identify a mechanism which might be playing the key role in the reverse shear transition. We show here when the magnetic shear has the same sign as the second derivative of the parallel flow with respect to the radial coordinate, the ion temperature gradient (ITG) mode may become more unstable. On the other hand, when the magnetic shear has opposite sign to the second derivative of the parallel velocity, the ITG mode may be completely stabilized. This result, therefore, shows that it is the relative sign of the second radial derivative of the equilibrium parallel flow with respect to the magnetic shear which may be the key factor for the enhanced reverse shear transition. Experimental supports for this theory and its far-reaching consequences in the *sustainment* of the improved modes are discussed.

II. STABILITY ANALYSIS

We choose a simple model of the ITG-driven modes.[9] We adopt a two-fluid theory in a sheared slab geometry, $\mathbf{B} = B_o[\mathbf{z} + (x/L_s)\mathbf{y}]$, where L_s is the scale length of magnetic shear. The x, y, and z directions in the sheared slab geometry are defined as the radial, poloidal, and toroidal directions in the tokamak configuration. We assume a background plasma with all inhomogeneities only in the radial direction, where perturbations have the form $\phi(\mathbf{x},t) = \phi(x)\exp[i(k_y y + k_z z - \omega t)]$. We ignore finite gyroradius effects by limiting consideration to the wavelength domain $k_\perp \rho_i \ll 1$, where ρ_i is the ion gyroradius. We then write the equations of continuity for ions:

$$\frac{\partial \tilde{n}_i}{\partial t} + \nabla.(n_0 + \tilde{n}_i)(v_0 + \tilde{v}_\perp + \tilde{v}_\parallel) = 0, \tag{1}$$

where

$$\hat{b} = \mathbf{B}/|B|, \quad P_i = P_{i0}(x) + \tilde{p}_i, \quad n_i = n_0 + \tilde{n}_i,$$

$$v_0 = v_{\parallel 0}(x)\hat{e}_\parallel,$$

$$\tilde{v}_\perp = v_E + v_{Di} + v_P,$$

$$v_E = \frac{c}{B}\hat{b} \times \nabla_\perp \phi,$$

$$v_{Di} = \frac{c}{eBn_i}\hat{b} \times \nabla_\perp P_i,$$

$$v_P = -\frac{c^2 m_i}{eB^2}\left(\frac{\partial}{\partial t} + (v_0 + v_E + v_{Di}).\nabla\right)\nabla_\perp \phi.$$

Assuming electron dynamics to be adiabatic Eq. (1) can be reduced to

$$\left(\frac{\partial}{\partial t} + v_0.\nabla\right)(1 - \nabla_\perp^2)\tilde{\phi} + v_D\left[1 + \left(\frac{1+\eta_i}{\tau}\right)\nabla_\perp^2\right]\nabla_y \tilde{\phi}$$
$$- \hat{b} \times \nabla\tilde{\phi}.\nabla(\nabla_\perp^2 \tilde{\phi}) + \nabla_\parallel \tilde{v}_\parallel = 0. \tag{2}$$

Here, in deriving Eq. (2) we have rescaled the quantities as $\tilde{\phi} \equiv e\phi/T_e$, $\tilde{v}_\parallel \equiv \tilde{v}_{\parallel i}/c_s$, $\tilde{p} \equiv [\tilde{p}_i/P_{i0}](T_i/T_e)$, $\tau \equiv T_e/T_i$, $\mathcal{Y} \equiv \Gamma/\tau$, $\mu \equiv \mu_\parallel \omega_{ci}/c_s^2$. Here, Γ is the ration of specific heats, and $\mu_\parallel$ is the parallel viscocity (due to either Landau damping or collisional viscosity) required for saturation of the turbulence.

Similarly, the parallel momentum equation for ions can be written as

$$\frac{\partial \tilde{v}_{\parallel i}}{\partial t} + (v_E + v_{\parallel 0}(x)).\nabla(v_{\parallel 0}(x) + \tilde{v}_{\parallel i})$$
$$= -\frac{e}{m_i}\nabla_\parallel \phi - \frac{1}{m_i n_i}\nabla_\parallel P_i + \mu_\parallel \nabla_\parallel^2 (v_0 + \tilde{v}_{\parallel i}) \tag{3}$$

and the rescaled equation as

$$\left(\frac{\partial}{\partial t} + v_0.\nabla\right)\tilde{v}_\parallel - \frac{v_0}{L_v}\nabla_y \tilde{\phi} + \hat{b} \times \nabla\tilde{\phi}.\nabla\tilde{v}_\parallel$$
$$= -\nabla_\parallel \tilde{\phi} - \nabla_\parallel \tilde{p} + \mu\nabla_\parallel^2 \tilde{v}_\parallel. \tag{4}$$

Finally, the equation of adiabatic pressure evolution is written as

$$\frac{\partial \tilde{p}_i}{\partial t} + v_E.\nabla P_{i0} + v_E.\nabla\tilde{p}_i + v_0.\nabla\tilde{p}_i + \Gamma P_{i0}\nabla_\parallel \tilde{v}_{\parallel i} = 0 \tag{5}$$

and the rescaled version as

$$\left(\frac{\partial}{\partial t} + v_0.\nabla\right)\tilde{p} + T_e v_D\left(\frac{1+\eta_i}{\tau}\right)\nabla_y \tilde{\phi} + \hat{b} \times \nabla_\perp \tilde{\phi}.\nabla\tilde{p}$$
$$+ \mathcal{Y}\nabla_\parallel \tilde{v}_\parallel = 0. \tag{6}$$

Here, all symbols are assumed to have the usual meaning unless otherwise stated explicitly. We make no attempt to speculate about the source of the parallel flow although a strongly peaked ion velocity parallel to the magnetic field is observed to coexist in tokamaks in the region where the plasma confinement is improved.[10] Parallel flow introduces a Doppler shift, $k_\parallel v_{\parallel o}(x)$, in all time derivatives and second, an extra term, $\mathbf{v}_E.\nabla v_{\parallel o}(x)$, representing radial convection of ion momentum. The second term makes the effect of parallel flow shear completely different from that of the perpendicular flow shear.[11]

To model the equilibrium parallel velocity we assume a simple general case of variation with the radial distance x:

$$v_0(x) = v_{00} + \frac{v_{00}}{L_{v1}}x + \frac{v_{00}}{2L_{v2}}x^2.$$

Linearizing Eqs. (2), (4), (6), and neglecting $\mathcal{Y}$ [which gives corrections of order $(k_\parallel/k_\perp)^4$], we write the eigenvalue equation as

$$\frac{\partial^2 \tilde{\phi}}{\partial x^2} + [\Lambda - Qx + Px^2] = 0, \tag{7}$$

where

$$\Lambda=-k_y^2+\frac{1-\Omega}{\Omega+K},\quad P=-\frac{J_2S}{\Omega(\Omega+K)}+\frac{S^2}{\Omega^2},$$

$$Q=\frac{J_1}{\Omega(\Omega+K)}S,\quad \Omega=\frac{\tilde{\omega}}{k_y v_D},\quad K=\frac{1+\eta_i}{\tau},\quad S=\frac{L_n}{L_s},$$

$$J_2=\left(\frac{v_{00}L_n}{L_{v2}}\right),\quad J_1=\left(\frac{v_{00}L_n}{L_{v1}}\right).$$

Equation (7) is a simple Weber equation. Depending on the sign of P, we have two types of solution. If $P<0$, i.e.,

$$\frac{J_2S}{\Omega(\Omega+K)}>\frac{S^2}{\Omega^2},$$

the solution which satisfies the physical boundary condition, i.e., $\phi\to 0$ at $x=\pm\infty$ is given by

$$\phi(x)=\phi_o\exp[-1/2\sqrt{|P|}(x-x_o)^2],\tag{8}$$

where $x_o=|Q|/2|P|$. The wave therefore does not propagate and is intrinsically undamped.

On the other hand, if $P>0$, Eq. (7) has the solution,

$$\phi(x)=\phi_o\exp[-i/2\sqrt{|P|}(x+x_o)^2].\tag{9}$$

So, in this case we have now a nonlocalized wave. The wave is therefore damped as in this case because of the convective wave energy leakage the perturbation will decay in time in the absence of any energy source feeding the wave.

From the above discussion it is clear that it is the parallel flow curvature which actually plays the key role in the stability of the mode. When the magnetic shear has the same sign as the parallel flow curvature, i.e., for positive magnetic shear ($L_s>0$), parallel flow curvature acts to destabilize the mode. On the other hand, for the negative magnetic shear configuration ($L_s<0$), i.e., when the magnetic shear has the opposite sign to the second derivative of the parallel flow with respect to the radial coordinate x, the parallel flow curvature acts to stabilize the mode. Flow curvature now forms an additional antiwell which pushes the wave function away from the mode rational surface, thereby enhancing stabilization.

The overall stability of the mode may also be obtained from the dispersion relation

$$\Lambda=\frac{Q^2}{4P}+i\sqrt{|P|}$$

which can be written more explicitly as

$$(1+k_y^2)\Omega^2+(K\kappa_y^2-1)\Omega =-\frac{J_1^2\Omega}{4(\Omega+K)\left(1-\frac{J_2\Omega}{S(\Omega+K)}\right)} -iS(\Omega+K)\sqrt{\left|1-\frac{J_2\Omega}{S(\Omega+K)}\right|}.\tag{10}$$

We will now solve the eigenvalue equation by using a numerical code developed by Bai *et al.*[12] For numerical solution we keep the contribution of $\mathcal{Y}$ when the eigenvalue equation reduces to

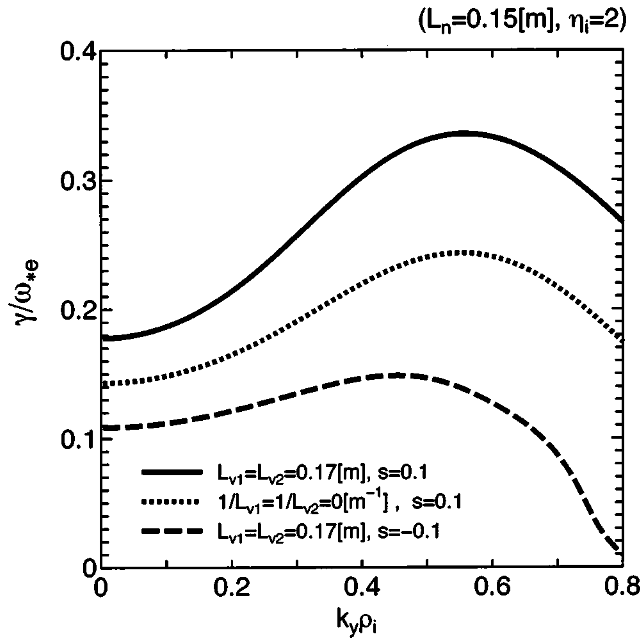

FIG. 1. Normalized growth rate of the ITG mode with $k_y\rho_i$ for negative and positive magnetic shear.

$$\frac{\partial^2\tilde{\phi}}{\partial x^2}+[\Lambda-Qx+P_mx^2]=0,\tag{11}$$

where

$$P_m=-\frac{J_2S}{\Omega(\Omega+K)}+\frac{S^2}{\Omega^2\left(1-\frac{\Gamma}{\tau}\frac{s^2x^2}{\Omega^2}\right)}.$$

It is clear that because of the x dependence of P_m, Eq. (11) is not solvable analytically. Figure 1 shows that for the case of positive magnetic shear the ITG mode is more destabilized than the case when there is no parallel flow. On the other hand, for the negative magnetic shear case the growth rate of the ITG mode decreases in the presence of parallel flow curvature and the mode can be fully stabilized for $k_y\rho_i>0.8$. Figure 2 shows the plot of real frequencies in these three cases. Figure 3 shows the variation of the ITG growth rate with the scale length of parallel flow curvature for the positive and negative magnetic shear cases. It reconfirms while for the positive shear case the growth rate of the ITG can be very high, in the case of negative shear the ITG mode can be completely stabilized by decreasing the scale length of the parallel flow curvature.

III. CONCLUSION

In summary, we present here a model for transition to the enhanced reverse shear (ERS) or negative central shear (NCS) mode triggered in tokamaks. Our studies show that when the magnetic shear has the same sign as the second derivative of the parallel velocity with respect to the radial coordinate, the ITG mode may become more unstable. On the other hand, when the magnetic shear has the opposite sign to the second derivative of the parallel velocity, the ITG mode may be completely stabilized. This result is similar to

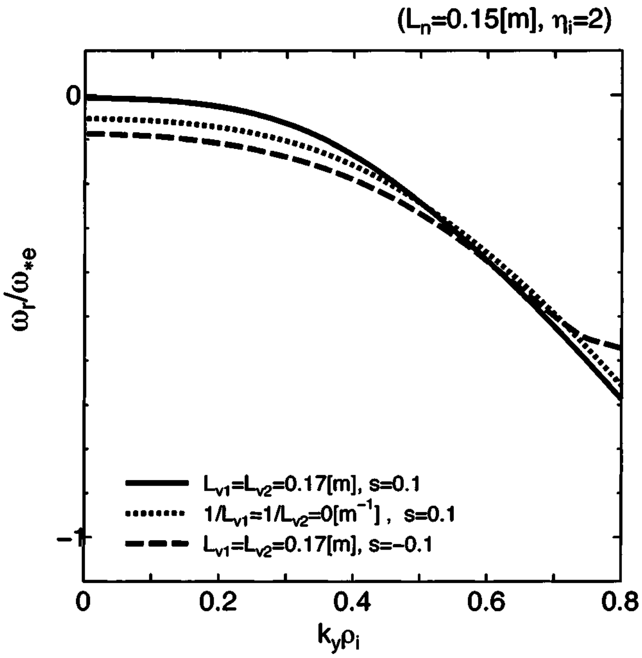

FIG. 2. Real part of the frequency of the ITG mode with $k_y\rho_i$ for negative and positive magnetic shear.

what we have found earlier for the PVS instabilities.[13,14] So, the similar result with the ITG mode, considered nowadays as the dominant source of anomalous energy losses in the low confinement (L) mode, therefore, shows on a firmer footing that it is the relative sign of the second radial derivative of the equilibrium parallel flow with respect to the magnetic shear which may be the key factor for the enhanced reverse shear transition.

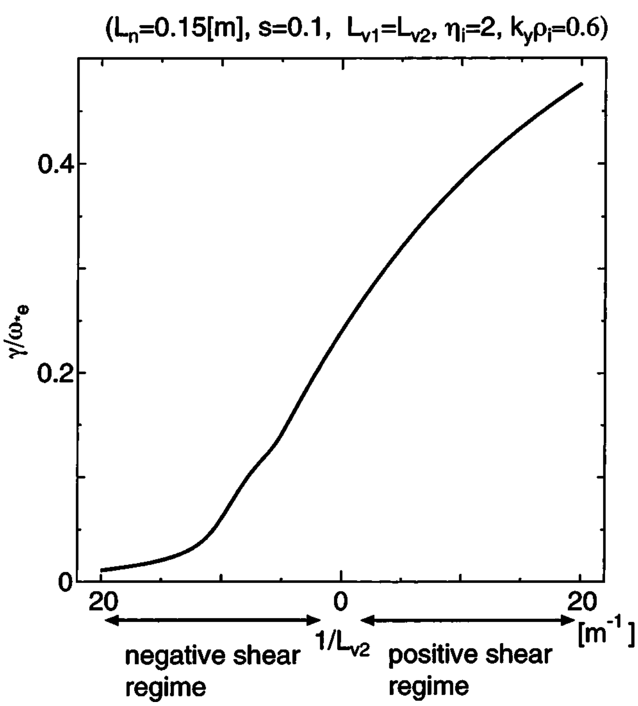

FIG. 3. Normalized growth rate of the ITG mode with L_{v2} for negative and positive magnetic shear.

Now, the real benefit of the outcome of this work is that it puts forward a novel idea of transport barrier formation. This is that, contrary to the usual notion that a parallel flow shear is always destabilizing,[15] the destabilizing influence of the shear in the parallel flow can be changed altogether if one takes the effect of the flow curvature into account. The transverse curvature in the parallel flow can overcome the destabilizing influence of the shear and can render the low frequency modes stable and can thereby reduce the radial transport. This new scenario that parallel flow can be a viable candidate for the stabilization of instabilities is very promising (note the usual picture is that it is the perpendicular $\mathbf{E}\times\mathbf{B}$ flow which does the stabilization). This is because, in a tokamak, the parallel velocity is very nearly equal to the toroidal velocity whereas the perpendicular velocity to the poloidal velocity. Now, the poloidal rotation in tokamak suffers from several disadvantages over toroidal rotation, most notably that poloidal flow is efficiently damped by magnetic pumping. Indeed, experimentally measured damping time of poloidal flows is of the order of the ion–ion collision time or less, and hence is much shorter than the damping time of toroidal flows. As a result, poloidal rotation dies away immediately after the beams [in the NBI heating] are turned off leaving the plasma rotation in the toroidal direction. Toroidal rotation, on the other hand, is dissipated only through the diffusive transport of momentum which is expected to reduce to low, neoclassical levels. *Stabilization by parallel flow, therefore, seems to offer much more attractive prospect for high performance tokamak operation*. However, in order to arrive at a definitive conclusion a full nonlinear analysis should be performed and this will be reported in a separate article.

On experimental front, recent results from the Joint European Torus have shown that the reduction of small-scale turbulence in optimized magnetic shear regimes is directly related to the existence of a strongly sheared toroidal velocity in the area of the internal transport barrier.[10] Furthermore, the clear evidence for the theory developed here comes from the STOR-M tokamak at Canada where *no* change in the radial electric field is observed during the transition to the improved modes indicating that it is the parallel component of the flow which might be playing the key role in the transition.[16] Finally, we note the work by Smolyakov[17] where a more consistent treatment of the ion polarization drift was given. However, for the sake of this problem where the main theme is the change in the growth rate due to the parallel flow curvature the absolute value of the growth rate is not relevant.

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